Lecture 24 - Dec. 3

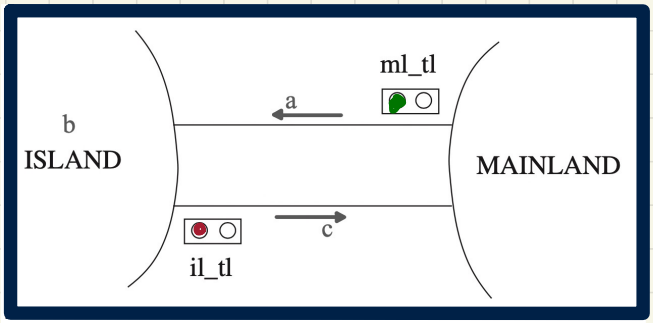
Bridge Controller

Adding Actions
Splitting Events
Preventing Livelock/Divergence
Proving Livelock/Divergence Freedom

Announcements/Reminders

- **Lab5** due today
- **Exam** review sessions and office hours TBA
- Sample exam questions to come
- Data Sheet:
  + Hand-Writing & Screenshots allowed
  + Font size requirement: ≥ 10pt

Fixing m2: Adding Actions



ML_tl_green
when
ml_tl = red
a + b < d
c = 0
then
ml_tl := green
il_tl := red
end

IL_tl_green
when
il_tl = red
b > 0
a = 0
then
il_tl := green
ml_tl := red
end

ml_tl' = g ∧ il_tl' = r

Added inv2_5 : | * b
new inv.
ML_arr ML_tl IL_arr IL_tl
ML_tl_green/inv2_5/INV
ml_tl = red ∨ il_tl = red

	axm0_1	$d \in \mathbb{N}$
	axm0_2	$d > 0$
	axm2_1	$COLOUR = \{green, red\}$
	axm2_2	$green \neq red$
m0	inv0_1	$n \in \mathbb{N}$
m0	inv0_2	$n \leq d$
	inv1_1	$a \in \mathbb{N}$
	inv1_2	$b \in \mathbb{N}$
	inv1_3	$c \in \mathbb{N}$
m1	inv1_4	$a + b + c = n$
m1	inv1_5	$a = 0 \vee c = 0$
	inv2_1	$ml\_tl \in COLOUR$
	inv2_2	$il\_tl \in COLOUR$
m2	inv2_3	$ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
m2	inv2_4	$il\_tl = green \Rightarrow b > 0 \wedge a = 0$
	inv2_5	$ml\_tl = red \vee il\_tl = red$

IL_tl = red

ML_tl_green
IL_tl_green.

Concrete grd. of ML_tl_gn.: ml_tl = red, a + b < d, c = 0

⊢ * green = red ∨ red = red.

green ml_tl' = red ∨ red il_tl' = red

Exercise: Specify IL_tl_green/inv2_5/INV

INV2_3 : ml_tl = g ⇒ a+b<d ∧ c=0

INV2_4 : tl_tl = g ⇒ b>0 ∧ a=0

Discussed (Thursday)

tl_tl = g ⇒ a=0

ML_out / INV2_4 / INV

IL_out / INV2_3 / INV

ml_tl = g ⇒ c=0

safety

To Discuss (Today)

ml_tl = g ⇒ a+b<d

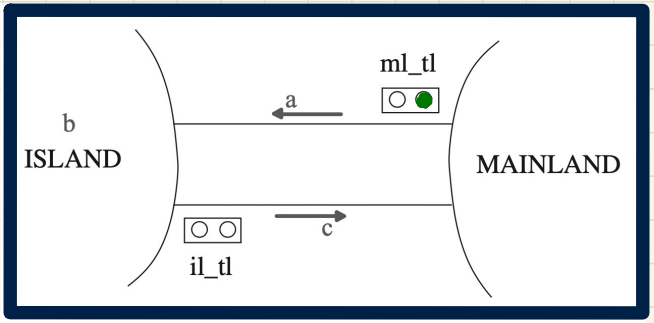
ML_out / INV2_3 / INV

IL_out / INV2_4 / INV

tl_tl = g ⇒ b>0

capacity

Invariant Preservation: ML_out/inv2_3/INV



ML_out
when $ml\_tl = green$
then $a := a + 1$
end

$a' = a + 1$

IL_out
when $il\_tl = green$
then $b := b - 1$
$c := c + 1$
end

variables:
a, b, c
ml_tl
il_tl

invariants:
inv2_1 : $ml\_tl \in COLOUR$
inv2_2 : $il\_tl \in COLOUR$
inv2_3 : $ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
inv2_4 : $il\_tl = green \Rightarrow b > 0 \wedge a = 0$

Exercise: Specify IL_out/inv2_4/INV

$b' = b - 1$
$b - 1 > 0$

ML_out/inv2_3/INV

axm0_1	$d \in \mathbb{N}$
axm0_2	$d > 0$
axm2_1	$COLOUR = \{green, red\}$
axm2_2	$green \neq red$
inv0_1	$n \in \mathbb{N}$
inv0_2	$n \leq d$
inv1_1	$a \in \mathbb{N}$
inv1_2	$b \in \mathbb{N}$
inv1_3	$c \in \mathbb{N}$
inv1_4	$a + b + c = n$
inv1_5	$a = 0 \vee c = 0$
inv2_1	$ml\_tl \in COLOUR$
inv2_2	$il\_tl \in COLOUR$
inv2_3	$ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
inv2_4	$il\_tl = green \Rightarrow b > 0 \wedge a = 0$
inv2_5	$ml\_tl = red \vee il\_tl = red$
Concrete guards of *ML_out*	$ml\_tl = green$
⊢	
Concrete invariant **inv2_3** with *ML_out*'s effect in the post-state	$ml\_tl = green \Rightarrow (a + 1) + b < d \wedge c = 0$

post-state of ML_out
$(a' = a + 1)$

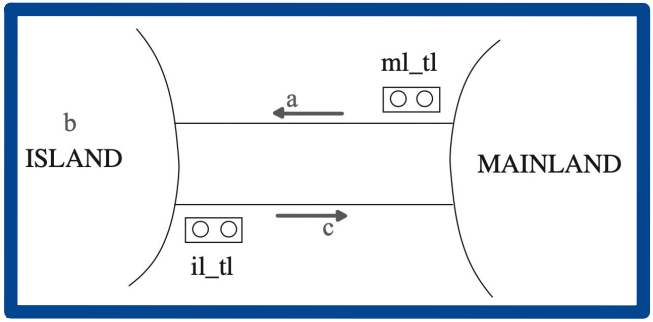
Discharging POs of m2: Invariant Preservation

First Attempt

$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \lor c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$ml\_tl = red \lor il\_tl = red$
$ml\_tl = green$
$\vdash$
$ml\_tl = green \Rightarrow (a + 1) + b < d \land c = 0$

MON

ML_out/inv2_3/INV



Unprovable

$a+b<d$
$c=0$
$ml\_tl = g.$
$\vdash (a+1)+b<d$

$$\frac{H \vdash P \qquad H \vdash Q}{H \vdash P \land Q}\ \text{AND\_R}$$

$$\frac{H, P, Q \vdash R}{H, P \land Q \vdash R}\ \text{AND\_L}$$

$$\frac{H, P \vdash Q}{H \vdash P \Rightarrow Q}\ \text{IMP\_R}$$

$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$\vdash$
$ml\_tl = green \Rightarrow (a + 1) + b < d \land c = 0$

IMP_R

$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$ml\_tl = green$
$\vdash$
$(a + 1) + b < d \land c = 0$

IMP_R ↓ IMP_L.

$a + b < d \land c = 0$
$ml\_tl = green$
$\vdash$
$(a + 1) + b < d \land c = 0$

AND_L

$a + b < d$
$c = 0$
$ml\_tl = green$
$\vdash$
$(a + 1) + b < d \land c = 0$

AND_R

$a + b < d$
$c = 0$
$ml\_tl = green$
$\vdash$
$(a + 1) + b < d$

??

$a + b < d$
$c = 0$
$ml\_tl = green$
$\vdash$
$c = 0$

HYP

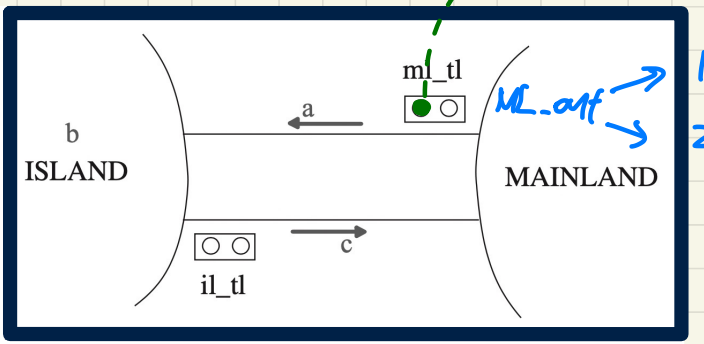
Understanding the Failed Proof on INV

variables:
a, b, c
ml_tl
il_tl

invariants:
inv2_1 : $ml\_tl \in COLOUR$
inv2_2 : $il\_tl \in COLOUR$
inv2_3 : $ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
inv2_4 : $il\_tl = green \Rightarrow b > 0 \wedge a = 0$

ML_out
when $ml\_tl = green$
then $a := a + 1$
end

IL_out
when $il\_tl = green$
then $b := b - 1$ $c := c + 1$
end



Unprovable Sequent:

$$a + b < d$$
$$\wedge \quad c = 0$$
$$\wedge \quad ml\_tl = green$$
$$\vdash$$
$$(a + 1) + b < d$$

for $x+1<y$, $x < y \Rightarrow ? \; x+1 < y$ — not true in general

x can't be equal to $y-1$!

e.g. $3+1 \not< 4$

$d = 3, b = 0, a = 0$	$[(a+1)+b < d$ evaluates to ***true*** $]$
$d = 3, b = 1, a = 0$	$[(a+1)+b < d$ evaluates to ***true*** $]$
$d = 3, b = 0, a = 1$	$[(a+1)+b < d$ evaluates to ***true*** $]$
$d = 3, b = 0, a = 2$	$[(a+1)+b < d$ evaluates to ***false*** $]$
$d = 3, b = 1, a = 1$	$[(a+1)+b < d$ evaluates to ***false*** $]$
$d = 3, b = 2, a = 0$	$[(a+1)+b < d$ evaluates to ***false*** $]$

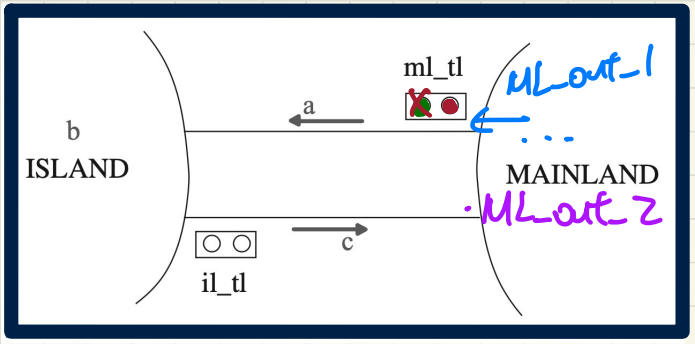
$(a+1)+b \neq d$ ↳ no need to turn ml_tl to red right away.

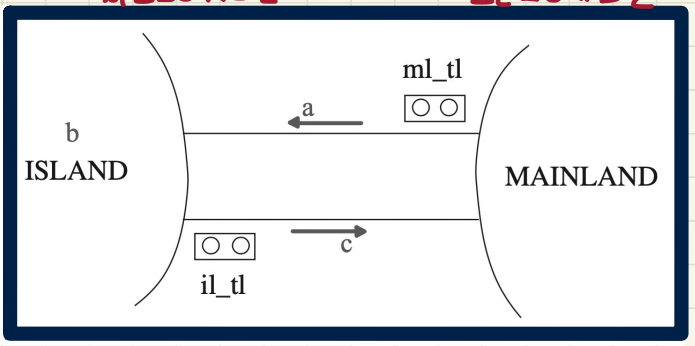
ML_ant allowing one now car on the bridge

$(a+1)+b = d$ need to turn ml_tl to red right away.

Fixing m2: Splitting Events

m1: ML_out
m2: ML_out_1, ML_out_2
IL_out
IL_out_1, IL_out_2





ML_out_1
when
ml_tl = green
a + b + 1 ≠ d
then
a := a + 1
end

ML_out_2
when
ml_tl = green
a + b + 1 = d
then
a := a + 1
ml_tl := red
end

disable ML_out_1

IL_out_1
when
il_tl = green
b ≠ 1
then
b := b − 1
c := c + 1
end

IL_out_2
when
il_tl = green
b = 1
then
b := b − 1
c := c + 1
il_tl := red
end

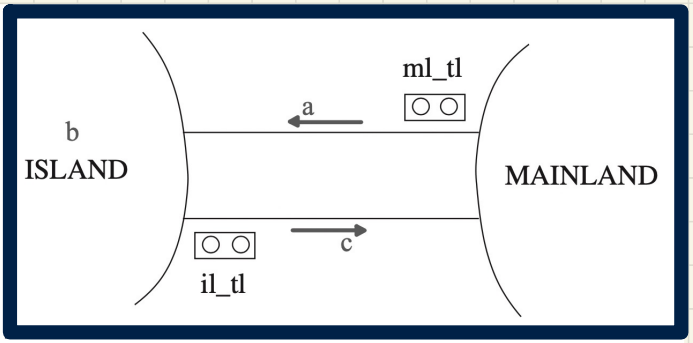
as soon as the capacity limit is reached, turn ml_tl to red

Current m2 May Livelock

infinite interleaving of new events

ML_tl_green
when
$ml\_tl = red$
$a + b < d$
$c = 0$
then
$ml\_tl := green$
$il\_tl := red$
end

IL_tl_green
when
$il\_tl = red$
$b > 0$
$a = 0$
then
$il\_tl := green$
$ml\_tl := red$
end



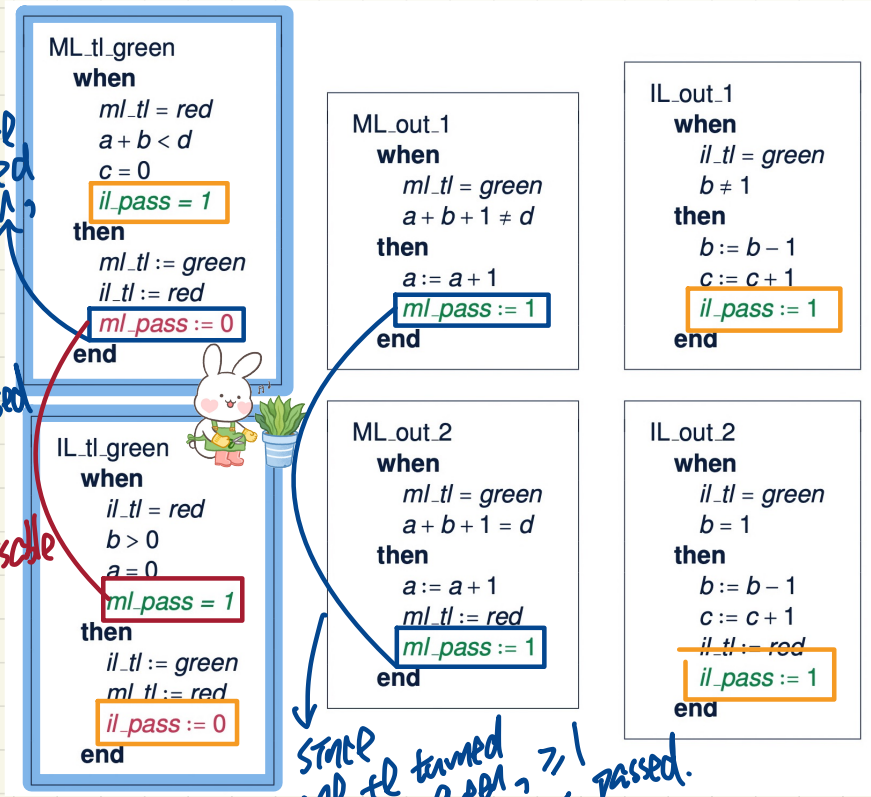
The current m2 diverges ∵ there's one valid trace of infinite interleaving of new events

starting point of livelock

⟨ *init*	, *ML_tl_green*	, *ML_out_1*	, *IL_in*	, *IL_tl_green*	, *ML_tl_green*	, *IL_tl_green*	, . . . ⟩
$d = 2$ $a' = 0$ $b' = 0$ $c' = 0$ $ml\_tl = red$ $il\_tl = red$	$d = 2$ $a' = 0$ $b' = 0$ $c' = 0$ $ml\_tl' = green$ $il\_tl' = red$	$d = 2$ $a' = 1$ $b' = 0$ $c' = 0$ $ml\_tl' = green$ $il\_tl' = red$	$d = 2$ $a' = 0$ $b' = 1$ $c' = 0$ $ml\_tl' = green$ $il\_tl' = red$	$d = 2$ $a' = 0$ $b' = 1$ $c' = 0$ $ml\_tl' = red$ $il\_tl' = green$	$d = 2$ $a' = 0$ $b' = 1$ $c' = 0$ $ml\_tl' = green$ $il\_tl' = red$	$d = 2$ $a' = 0$ $b' = 1$ $c' = 0$ $ml\_tl' = red$ $il\_tl' = green$	

Fixing m2: Regulating Traffic Light Changes

Divergence Trace: <init, ML_tl_green, ML_out_1, IL_in, IL_tl_green, ML_tl_green, IL_tl_green, ...>



disabled IL_tl_green both new events enabled.

d = 2	ml_pass	il_pass
< init,	1	1
ML_tl_green,	0	1
ML_out_1,	1	1
ML_out_2,	1	1
IL_in,	1	1
IL_in,	1	1
IL_tl_green,	1	0
IL_out_1,	1	1
IL_out_2,	1	1
ML_in,	1	1
ML_in	1	1
>		

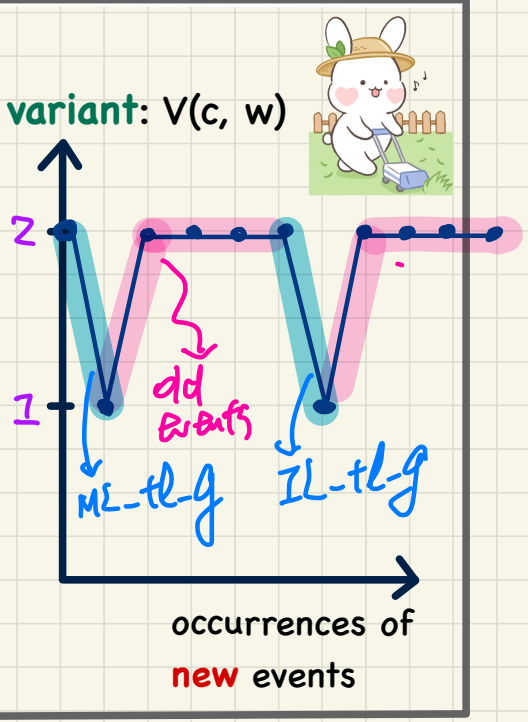
Fixing m2: Measuring Traffic Light Changes

ML_tl_green
when
ml_tl = red
a + b < d
c = 0
il_pass = 1
then
ml_tl := green
il_tl := red
ml_pass := 0
end

IL_tl_green
when
il_tl = red
b > 0
a = 0
ml_pass = 1
then
il_tl := green
ml_tl := red
il_pass := 0
end

d = 2	ml_pass	il_pass	
< init,	1	1	2
ML_tl_green,	0	1	1
ML_out_1,	1	1	2
ML_out_2,	1	1	2
IL_in,	1	1	2
IL_in,	1	1	2
IL_tl_green,	1	0	1
IL_out_1,	1	1	2
IL_out_2,	1	1	2
ML_in,	1	1	2
ML_in	1	1	2
>			

variants: *ml_pass + il_pass*



PO of Convergence/Non-Divergence/Livelock Freedom

A New Event Occurrence Decreases Variant

$$\frac{\begin{array}{l} A(c) \\ I(c,v) \\ J(c,v,w) \\ H(c,w) \\ \vdash \\ V(c,F(c,w)) < V(c,w) \end{array}}{\text{VAR}}$$

✓ post-state value of var.

pre-state value of var.

Variants: ml_pass + il_pass

ML_tl_green/VAR

ML_tl_green
  when
    $ml\_tl = red$
    $a + b < d$
    $c = 0$
    $il\_pass = 1$
  then
    $ml\_tl := green$
    $il\_tl := red$
    $ml\_pass := 0$
  end

$ml\_pass' = 0$
$\wedge\ tl\_pass' = tl\_pass$

$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \vee c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
$il\_tl = green \Rightarrow b > 0 \wedge a = 0$
$ml\_tl = red \vee il\_tl = red$
$ml\_pass \in \{0, 1\}$
$il\_pass \in \{0, 1\}$
$ml\_tl = red \Rightarrow ml\_pass = 1$
$il\_tl = red \Rightarrow il\_pass = 1$
$ml\_tl = red$
$a + b < d$
$il\_pass = 1$
$\vdash$
$0 \leq il\_pass < ml\_pass + il\_pass$

$* \quad ml\_pass' + tl\_pass' < ml\_pass + tl\_pass$

PO of Relative Deadlock Freedom

Abstract m1

variables: a, b, c

ML_out
when
$a + b < d$
$c = 0$
then
$a := a + 1$
end

ML_in
when
$c > 0$
then
$c := c - 1$
end

IL_in
when
$a > 0$
then
$a := a - 1$
$b := b + 1$
end

IL_out
when
$b > 0$
$a = 0$
then
$b := b - 1$
$c := c + 1$
end

axm0_1 $d \in \mathbb{N}$
axm0_2 $d > 0$
axm2_1 $COLOUR = \{green, red\}$
axm2_2 $green \neq red$
inv0_1 $n \in \mathbb{N}$
inv0_2 $n \leq d$
inv1_1 $a \in \mathbb{N}$
inv1_2 $b \in \mathbb{N}$
inv1_3 $c \in \mathbb{N}$
inv1_4 $a + b + c = n$
inv1_5 $a = 0 \lor c = 0$
inv2_1 $ml\_tl \in COLOUR$
inv2_2 $il\_tl \in COLOUR$
inv2_3 $ml\_tl = green \Rightarrow a + b < d \land c = 0$
inv2_4 $il\_tl = green \Rightarrow b > 0 \land a = 0$
inv2_5 $ml\_tl = red \lor il\_tl = red$
inv2_6 $ml\_pass \in \{0, 1\}$
inv2_7 $il\_pass \in \{0, 1\}$
inv2_8 $ml\_tl = red \Rightarrow ml\_pass = 1$
inv2_9 $il\_tl = red \Rightarrow il\_pass = 1$

Disjunction of *abstract* guards

$a + b < d \land c = 0$ } guards of *ML_out* in m_1
$\lor$ $c > 0$ } guards of *ML_in* in m_1
$\lor$ $a > 0$ } guards of *IL_in* in m_1
$\lor$ $b > 0 \land a = 0$ } guards of *IL_out* in m_1

$\vdash$

Disjunction of *concrete* guards

$ml\_tl = red \land a + b < d \land c = 0 \land il\_pass = 1$ } guards of *ML_tl_green* in m_2
$\lor$ $il\_tl = red \land b > 0 \land a = 0 \land ml\_pass = 1$ } guards of *IL_tl_green* in m_2
$\lor$ $ml\_tl = green \land a + b + 1 \neq d$ } guards of *ML_out_1* in m_2
$\lor$ $ml\_tl = green \land a + b + 1 = d$ } guards of *ML_out_2* in m_2
$\lor$ $il\_tl = green \land b \neq 1$ } guards of *IL_out_1* in m_2
$\lor$ $il\_tl = green \land b = 1$ } guards of *IL_out_2* in m_2
$\lor$ $a > 0$ } guards of *ML_in* in m_2
$\lor$ $c > 0$ } guards of *IL_in* in m_2

Concrete m2

ML_tl_green
when
$ml\_tl = red$
$a + b < d$
$c = 0$
$il\_pass = 1$
then
$ml\_tl := green$
$il\_tl := red$
$ml\_pass := 0$
end

IL_tl_green
when
$il\_tl = red$
$b > 0$
$a = 0$
$ml\_pass = 1$
then
$il\_tl := green$
$ml\_tl := red$
$il\_pass := 0$
end

ML_out_1
when
$ml\_tl = green$
$a + b + 1 \neq d$
then
$a := a + 1$
$ml\_pass := 1$
end

IL_out_1
when
$il\_tl = green$
$b \neq 1$
then
$b := b - 1$
$c := c + 1$
$il\_pass := 1$
end

ML_out_2
when
$ml\_tl = green$
$a + b + 1 = d$
then
$a := a + 1$
$ml\_tl := red$
$ml\_pass := 1$
end

IL_out_2
when
$il\_tl = green$
$b = 1$
then
$b := b - 1$
$c := c + 1$
$il\_tl := red$
$il\_pass := 1$
end

IL_in
when
$a > 0$
then
$a := a - 1$
$b := b + 1$
end

ML_in
when
$c > 0$
then
$c := c - 1$
end

Discharging POs of m2: Relative Deadlock Freedom

$d \in \mathbb{N}$
$d > 0$
$COLOUR = \{green, red\}$
$green \neq red$
$n \in \mathbb{N}$
$n \leq d$
$a \in \mathbb{N}$
$b \in \mathbb{N}$
$c \in \mathbb{N}$
$a + b + c = n$
$a = 0 \lor c = 0$
$ml\_tl \in COLOUR$
$il\_tl \in COLOUR$
$ml\_tl = green \Rightarrow a + b < d \land c = 0$
$il\_tl = green \Rightarrow b > 0 \land a = 0$
$ml\_tl = red \lor il\_tl = red$
$ml\_pass \in \{0, 1\}$
$il\_pass \in \{0, 1\}$
$ml\_tl = red \Rightarrow ml\_pass = 1$
$il\_tl = red \Rightarrow il\_pass = 1$
$a + b < d \land c = 0$
$\lor\ c > 0$
$\lor\ a > 0$
$\lor\ b > 0 \land a = 0$
$\vdash$
$ml\_tl = red \land a + b < d \land c = 0 \land il\_pass = 1$
$\lor\ il\_tl = red \land b > 0 \land a = 0 \land ml\_pass = 1$
$\lor\ ml\_tl = green$
$\lor\ il\_tl = green$
$\lor\ a > 0$
$\lor\ c > 0$

IS#1

$d \in \mathbb{N}$
$d > 0$
$b \in \mathbb{N}$
$ml\_tl = red$
$il\_tl = red$
$ml\_tl = red \Rightarrow ml\_pass = 1$
$il\_tl = red \Rightarrow il\_pass = 1$
$\vdash$
$b < d \land ml\_pass = 1 \land il\_pass = 1$
$\lor\ b > 0 \land ml\_pass = 1 \land il\_pass = 1$

IS#2

$d \in \mathbb{N}$
$d > 0$
$b \in \mathbb{N}$
$ml\_tl = red$
$il\_tl = red$
$ml\_pass = 1$
$il\_pass = 1$
$\vdash$
$b < d \land ml\_pass = 1 \land il\_pass = 1$
$\lor\ b > 0 \land ml\_pass = 1 \land il\_pass = 1$

IS#3

$d > 0$
$b \in \mathbb{N}$
$\vdash$
$b < d \lor b > 0$

ARI

$d > 0$
$b > 0 \lor b = 0$
$\vdash$
$b < d \lor b > 0$

OR_L

$d > 0$
$b > 0$
$\vdash$
$b < d \lor b > 0$

OR_R2

$d > 0$
$b > 0$
$\vdash$
$b > 0$

HYP

$d > 0$
$b = 0$
$\vdash$
$b < d \lor b > 0$

EQ_LR_MON

$d > 0$
$\vdash$
$0 < d \lor 0 > 0$

OR_R1

$d > 0$
$\vdash$
$0 < d$

HYP

1st Refinement and 2nd Refinement: Provably Correct

Abstract m1

constants: d

axioms:
- **axm0_1** : $d \in \mathbb{N}$
- **axm0_2** : $d > 0$

variables: a, b, c

invariants:
- **inv1_1** : $a \in \mathbb{N}$
- **inv1_2** : $b \in \mathbb{N}$
- **inv1_3** : $c \in \mathbb{N}$
- **inv1_4** : $a + b + c = n$
- **inv1_5** : $a = 0 \vee c = 0$

variants: $2 \cdot a + b$

init
begin
$a := 0$
$b := 0$
$c := 0$
end

ML_out
when
$a + b < d$
$c = 0$
then
$a := a + 1$
end

ML_in
when
$c > 0$
then
$c := c - 1$
end

IL_in
when
$a > 0$
then
$a := a - 1$
$b := b + 1$
end

IL_out
when
$b > 0$
$a = 0$
then
$b := b - 1$
$c := c + 1$
end

Correctness Criteria:

+ Guard Strengthening
+ Invariant Establishment
+ Invariant Preservation
+ Convergence
+ Relative Deadlock Freedom

Concrete m2

constants: d

sets: $COLOR$

axioms:
- **axm0_1** : $d \in \mathbb{N}$
- **axm0_2** : $d > 0$
- **axm2_1** : $COLOR = \{green, red\}$
- **axm2_2** : $green \neq red$

variables:
a
b
c
ml_tl
il_tl
ml_pass
il_pass

invariants:
- **inv2_1** : $ml\_tl \in COLOUR$
- **inv2_2** : $il\_tl \in COLOUR$
- **inv2_3** : $ml\_tl = green \Rightarrow a + b < d \wedge c = 0$
- **inv2_4** : $il\_tl = green \Rightarrow b > 0 \wedge a = 0$
- **inv2_5** : $ml\_tl = red \vee il\_tl = red$
- **inv2_6** : $ml\_pass \in \{0, 1\}$
- **inv2_7** : $il\_pass \in \{0, 1\}$
- **inv2_8** : $ml\_tl = red \Rightarrow ml\_pass = 1$
- **inv2_9** : $il\_tl = red \Rightarrow il\_pass = 1$

variants: $ml\_pass + il\_pass$

ML_tl_green
when
$ml\_tl = red$
$a + b < d$
$c = 0$
$il\_pass = 1$
then
$ml\_tl := green$
$il\_tl := red$
$ml\_pass := 0$
end

IL_tl_green
when
$il\_tl = red$
$b > 0$
$a = 0$
$ml\_pass = 1$
then
$il\_tl := green$
$ml\_tl := red$
$il\_pass := 0$
end

ML_out_1
when
$ml\_tl = green$
$a + b + 1 \neq d$
then
$a := a + 1$
$ml\_pass := 1$
end

ML_out_2
when
$ml\_tl = green$
$a + b + 1 = d$
then
$a := a + 1$
$ml\_tl := red$
$ml\_pass := 1$
end

IL_out_1
when
$il\_tl = green$
$b \neq 1$
then
$b := b - 1$
$c := c + 1$
$il\_pass := 1$
end

IL_out_2
when
$il\_tl = green$
$b = 1$
then
$b := b - 1$
$c := c + 1$
$il\_tl := red$
$il\_pass := 1$
end

ML_in
when
$c > 0$
then
$c := c - 1$
end

IL_in
when
$a > 0$
then
$a := a - 1$
$b := b + 1$
end